Credit BuVaR: Asymmetric spread VaR with default

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Abstract A tradeable credit instrument shows two forms of credit risk — a continuous spread risk and a discontinuous default risk. The Basel market risk framework requires the two risks to be modelled separately for the purpose of regulatory capital but this gives rise to issues of risk aggregation. The author proposes a risk metric called credit bubble VaR (Cr. buVaR) that combines these dual risks under a common historical simulation value-at-risk (VaR) approach. By using a single model, Cr. buVaR bypasses the problem of risk aggregation. Credit risks can then be aggregated with market risk in a diversifiable manner. Cr. buVaR is also found to be forward-looking with respect to issuer credit default and is not procyclical. The model is motivated by evidence from the 2008 crisis that issuer defaults and spread movements exhibit asymmetry, and that defaults are always preceded by rapid spread widening. The method involves scaling the positive side of the return distribution of credit spreads in proportion to current spread levels. By drawing inferences from studies on the ‘credit spread puzzle’, it is deduced that the incremental loss of Cr. buVaR over spread VaR is due to default risk.

Keywords: value at risk, credit risk, procyclicality, extreme events, countercyclical capital, credit risk aggregation

INTRODUCTION
Credit risk refers to the risk that a company is unable to pay its debt obligations, leading to a bankruptcy. Debt securities issued by the company provide price discovery for this credit risk in the form of observed credit spreads in the secondary market. Financial institutions which hold a portfolio of such securities or their derivatives are exposed to several types of risks — the continuous movements of traded spreads, the possibility of a default (a discontinuous event risk), and the risk of correlated defaults among the issuers in the portfolio. This paper pertains to tradable credit instruments and will exclude loans to companies.
Consider a bank which holds a portfolio of bonds and credit default swaps (CDS) written on many issuers. Basel regulation requires that the risks mentioned be modelled using internal models, specifically value-at-risk (VaR) models. VaR of confidence level \( q \), is the \((1 – q) \) quantile of the profit and loss (P&L) distribution of the portfolio, estimated over a time horizon \( T \). A large \( q \) is chosen so that the quantile represents extreme tail loss.

A common way to model the spread risk, where data is plentiful, is to use an historical simulation VaR approach. At day \( t \), a sample of simulated returns (often called scenarios) is derived from the spread variable \( X \) over a past observation period. In this paper, for illustration purposes, a one-year period consisting of 250 trading days will be used. The return vector is \( \mathbf{R}_n = \ln(\mathbf{X}_n/\mathbf{X}_{n-1}) \) where \( n = t, t – 1, \ldots, t – 249 \). Each position in the portfolio is mapped to benchmark issuer risk factors; if the portfolio contains \( N \) risk factors, then there are \( N \) return vectors. The combination of return vectors is used to derive a distribution of P&L for the portfolio. This will be detailed later. For the purpose of regulatory capital, Basel requires the VaR of this distribution at \( q = 99 \) per cent and \( T = 10 \) days — this is called spread VaR.

To model default risks, since a company can only default once, it follows that a company will not have an observable history of defaults to afford the use of an historical simulation approach. Instead, Monte Carlo simulation is often used to generate hypothetical occurrences of ratings migration (of which default is a special case) at the horizon. Briefly speaking, the simulation assumes that the driver of credit default follows a particular distribution; the simulated number is then mapped to an end-state rating. The probabilities of migration from a current rating to various end-state ratings (the worst state being default) are contained in the rating transition matrix, whose elements are one-year transition rates compiled by rating agencies using actual past statistics of defaults and upgrades/downgrades in a particular benchmark sector. All the positions in the portfolio are mapped to relevant sectors before simulation. The correlation risk between various sectors is modelled using correlated random numbers, where the correlation coefficients are estimated separately. Suppose a B-rated bond migrates to a particular end state, BB, during a simulation, the bond is revalued using a BB-credit curve for that sector. The P&L caused by this move is stored and the simulation is repeated many times, to give a P&L distribution. For the purposes of regulatory capital, Basel requires the VaR of this distribution at \( q = 99.9 \) per cent and \( T = 1 \) year — this is called Credit VaR. Several popular models exist, as discussed by Crouhy et al.\(^1\)

A serious weakness of credit models is that default rates used in transition matrices are known to be very late in reflecting changing market conditions since they are based on backward-looking default statistics. Indeed, the credit crisis has led to criticism that the use of credit models for capital buffers exacerbated the crisis. Since rating revisions by analysts often trail market events and default rates are reflective of the past (not the future), credit models understate risk in the boom stage in the run-up to a crisis, and overstate risk during a downturn.
A capital framework linked to such models will encourage a buildup of financial bubbles and systemic unwinding of positions in a subsequent crisis. The Turner Review\(^2\) highlighted this danger of procyclicality and calls for a reformed risk measure which is countercyclical — providing a thicker capital buffer during good times that could be drawn down during crises.

A second weakness is that the disjointed approach to modelling spread risk and default risk makes aggregation difficult. Currently, the two are simply added for capital purposes, without accounting for diversification. Furthermore, it is not meaningful to add loss quantiles of different confidence levels and time horizons. The same disjointed approach and aggregation problem exists between credit and market risks, even though both risks are clearly intertwined within a particular instrument. See, for example, the BIS working paper\(^3\) which discusses the interaction of these risks and the modelling challenges of aggregation.

This paper proposes a unified approach which could overcome the two weaknesses mentioned. It uses only credit spread data in an historical simulation VaR setting and does not rely on transition matrices or ratings information.

**EVIDENCE FROM THE CREDIT CRISIS**

The model is motivated by two observations from the 2008 credit crisis.

First, actual credit defaults/stresses revealed that defaults are not simply jumps, as modelled in academia. The problem with jump–diffusion models in general is that defaults are modelled as Poisson arrivals independent of recent market conditions. Thus, it will not preempt imminent defaults even though it provides a realistic skewed/fat-tailed distribution. Contrast Figure 1, selected issuers that were impaired in the 2008 crisis, with Figure 2, a stylised jump diffusion process — their phenomenology is patently different. Actual defaults are always preceded by rapid spread widening (RSW) which acts as a leading signal, often weeks to months earlier. This observation suggests that a credit model conditional on RSW could be forward-looking.

**Figure 1:** Selected issuers that were impaired during the 2008 credit crisis

Note: Actual credit defaults/deterioration are not ‘jumps’ but are marked by rapid spread escalations which can be detected weeks ahead of potential default.

**Figure 2:** Stylised simulated mean reversion process with jumps

Note: The jumps occur as Poisson arrivals over time in the distribution.
Secondly, credit default and deterioration are asymmetric — spreads can only escalate upwards and a default only hurts long credit risk positions (i.e. long bonds and short default swaps). This obvious fundamental asymmetry is not reflected by spread VaR models, which measure only volatility of spreads and ignore other dynamics such as RSW. Consequently, spread VaR understates risks and, because it is estimated using a rolling observation sample, also lags significant market moves. The 2008 crisis showed that most of the trading book losses suffered by banks came from spread risk (which models have underestimated) rather than from actual defaults.

This paper improves the spread VaR model by incorporating default risk; it is motivated by the observed phenomena of RSW and risk asymmetry. The model, called credit bubble VaR (Cr. buVaR), is conditional on the spread level and is found to be forward-looking.

**CREDIT BUVAR MODEL**

An issuer’s credit spread is possibly the most direct and forward-looking market indicator of its default probability — the harbinger of default is RSW. It is proposed that the key to including default risks into spread VaR is to detect the occurrence of RSW and to penalise spread VaR proportionally using an adjustment \( \Delta_+ \) called the inflator. Long credit positions will incur losses only if spread changes are positive, whereas short credit positions will incur losses only if spread changes are negative. Since default can only hurt long positions (and never short ones), the inflator is applied only to the positive side of the distribution (i.e. to positive changes) to account for default risk.

Mathematically, on day \( t \), the raw return variable \( R_n \) undergoes a transformation:

\[
R'_n = \begin{cases} 
\Delta_+ R_n & \text{if } \text{sign}(R_n) > 0 \\
R_n & \text{if } \text{sign}(R_n) \leq 0 
\end{cases}
\]

where \( \Delta_+ \) (\( \geq 1 \)) is the inflator and \( n \) is the scenario number in the historical simulation VaR approach. As in most VaR methods, each position in the portfolio needs to be mapped to benchmark instruments — in this case, they are par bonds of different standard maturities of a particular issuer. In practice, the credit spreads of these benchmark instruments are taken from the more liquid CDS markets which give better price discovery.

Since the author wishes to penalise spread VaR rapidly as RSW manifests, a reasonable functional form of the inflator is:

\[
\Delta_+ = \exp(\omega_1 S^{\omega_2})
\]

where \( S \) is the current (benchmark) CDS spread, \( \omega_1 \) and \( \omega_2 \) are free parameters. The inflator needs to be capped, since spreads cannot widen indefinitely — at some upper limit of spread, \( S_{\text{cap}} \), the benchmark bond will be in default. This can be calculated from the pricing function of a bond (similar to those available in an Excel spreadsheet) where yield is a function of:

\[
\text{Yield} (\text{today, maturity, coupon rate, bond price, redemption price, coupon frequency}).
\]

Using the Lehman bankruptcy as a guide, the author assumes a recovery rate in the event of a default of about 10 per cent. It is also assumed that most bonds
are issued close to par (or 100) when the outlook of the issuer company is relatively attractive, such that the coupons are set at (or near) the risk free rate, so that the simplifying assumption that the coupon is the current risk free rate can be made. To be consistent with the CDS quotes, \( S \), the author also assumes that the bonds give quarterly cashflows just like a CDS. It will be shown later that in practical situations, such assumptions are reasonably acceptable. Under these assumptions, the spread of the benchmark bond at the point of default is given by:

\[
S_{\text{cap}} = Y_{\text{defaulted}} - \text{risk free rate} \quad (4)
\]

where \( Y_{\text{defaulted}} \) is the yield for the bond at the point of default. As an example, consider a five-year benchmark bond where the five-year risk free rate is 2 per cent, then:

\[
S_{\text{cap}} = \text{Yield (today, today} + 5 \times 365, 0.02, 10, 100, 4) - 0.02
\]

\[
= 5569 \text{ basis points.}
\]

The maximum inflator \( \Delta_{\text{max}} \) is defined as the adjustment that will inflate two standard deviations of spread returns (or 97.7 per cent VaR under a normal distribution) up to the percentage loss at the point of default represented by \( S_{\text{cap}} \). A two standard deviation move is chosen for convenience because variance is easy to estimate using historical data:

\[
\frac{S_{\text{cap}}}{S} - 1 = 2\sigma \Delta_{\text{max}} \quad (5)
\]

where \( S_{\text{cap}} \) also satisfies (2), hence:

\[
\Delta_{\text{max}} = \exp(\omega_1 S_{\text{cap}}^{\omega_2}) \quad (6)
\]

Substitution into (2) gives the response function:

\[
\Delta_+ = \exp\left\{ \left( \frac{S}{S_{\text{cap}}} \right)^{\omega_2} \times \ln\left( \frac{S_{\text{cap}}}{S} - 1 \right) / (2\sigma) \right\} \quad (7)
\]

which still contains one free parameter, \( \omega_2 \), that will be calibrated later. Table 1 illustrates the calculation of inflators for various standard maturities for an issuer using \( \omega_2 = 0.5 \). The scalar \( \Delta_+ \) is multiplied onto every positive return scenario of the sample distribution, as per (1), after which the transformed \( R_n' \), are used to value the portfolio of

Table 1: Derivation of inflators for various USD-benchmark bonds using a simple pricing function

| Recovery rate | 10 |
| Par           | 100 |
| \( \omega_2 \) | 0.5 |
| Pricing date  | 8 May 2011 |

<table>
<thead>
<tr>
<th>Benchmark maturity</th>
<th>1 year</th>
<th>2 years</th>
<th>3 years</th>
<th>5 years</th>
<th>7 years</th>
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<tr>
<td>Maturity</td>
<td>7 May 2012</td>
<td>7 May 2013</td>
<td>7 May 2014</td>
<td>6 May 2016</td>
<td>6 May 2018</td>
</tr>
<tr>
<td>Risk free rate (USD)</td>
<td>2.57%</td>
<td>2.57%</td>
<td>2.57%</td>
<td>2.57%</td>
<td>2.57%</td>
</tr>
<tr>
<td>Defaulted yield-to-maturity</td>
<td>325.80%</td>
<td>145.53%</td>
<td>96.29%</td>
<td>60.47%</td>
<td>46.20%</td>
</tr>
<tr>
<td>Defaulted (CDS) spread/ Bp</td>
<td>32,323</td>
<td>14,296</td>
<td>9,372</td>
<td>5,790</td>
<td>4,363</td>
</tr>
<tr>
<td>2 Standard Dev. of returns</td>
<td>0.1</td>
<td>0.12</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>Current CDS Spread (S)/Bp</td>
<td>100</td>
<td>110</td>
<td>120</td>
<td>125</td>
<td>140</td>
</tr>
<tr>
<td>Inflator</td>
<td>1.57</td>
<td>1.84</td>
<td>2.04</td>
<td>2.29</td>
<td>2.50</td>
</tr>
</tbody>
</table>
credit-risky products — more precisely, the portfolio is evaluated at ‘shifted’ levels \( X'_i \) whereby:

\[
X'_i = X_i \exp(R'_n)
\]

where scenario \( i = 1, 2, \ldots, 250 \) is the number of days into the past from day \(-t\) and \( n = t-i + 1 \). Using a univariate example, a bank’s portfolio is first evaluated using a product pricing function \( g(.) \) at the current state \( X_t \) giving value \( g(X_t) \). Then, for each scenario \(-i\) the portfolio is evaluated at state \( X'_i \) to give a value \( g(X'_i) \). The P&L vector at day \(-t\) is just the distribution of values \( \{g(X'_i) - g(X_t)\} \), where \( g(X'_i) \) is a 250-vector and \( g(X_t) \) is a scalar. Let the sample distribution of this P&L be \( y_n \).

Then, Cr. buVaR of confidence level \( q \) is defined as the \((1 - q)\) loss quantile of this ‘inflated’ P&L distribution \( y_n \) estimated over a one-day horizon. Alternatively, one can easily choose to use expected shortfall instead of quantile for the definition of Cr. buVaR.

Essentially, the model conjectures that the ‘all-in’ credit loss measure lies between spread VaR (a known underestimate) and the principal loss of the bond upon default (a logical upper bound) and it increases with spread level. The free parameter \( \omega_2 \) in the response function (7) determines the ‘speed’ of the increase and may be calibrated by the regulator.

Figure 3 shows that the inflator has a ‘hump’ shape — initially \( \Delta \) increases rapidly to penalise RSW, but after some point, it decreases to ensure that the inflated spread level never exceeds \( S_{\text{cap}} \), i.e., the bond holder does not lose more than the principal (less recovery value). In theory, when \( S_{\text{cap}} \) is reached the bond defaults and there is no longer any price uncertainty, hence VaR becomes zero. A lower \( \omega_2 \) gives a more conservative (higher) buVaR. The shaded zone is the typical trading range for spreads — for example, the high yield index, which represents lowly rated credits, traded below 700bp even during the credit crisis. Upon credit stress for specific issuers, spreads escalate rapidly to the right of the zone. From a regulatory perspective, a capital buffer should be imposed early before the market becomes disrupted, say before \( S \) goes above 1000. This argues for a more rapid response such as with \( \omega_2 = 0.5 \) in Figure 3. Figure 4 illustrates by how much VaR is expanded given an

\[
\text{Note: A plot of current spread, } S \text{ vs spread shifted by inflated VaR, } S(1 + 2\omega_2 \Delta). \]
inflator. The vertical difference between the two lines gives the buVaR loss in basis points.

The normal ‘working range’ for market risk supervision is where 5-year spreads are below 2000bp (or bond prices above 40 per cent of par). Once spreads have deteriorated beyond these levels, the market becomes disordered and banks normally remove the position from the trading book and manage it as a bad loan/impaired asset, ie outside the VaR framework. Within this ‘working range’, buVaR is reasonably robust (insensitive) to our earlier assumptions on coupons and recovery rates. Figure 5 shows the sensitivity to various coupon assumptions, and Figure 6 shows the sensitivity to recovery rates.

TEST RESULTS

The performance of Cr. buVaR vs spread VaR for long credit positions is compared. The test data set includes daily five-year CDS spreads until May 2009 for five issuers: Toyota, Philippines Government, Lehman, AIG and General Motors. All data are sourced from Markit. For simplicity, the P&Ls are not calculated using full valuation — a unit position (of either long five-year bonds or short five-year CDS protection) of the underlying risk factor is assumed, and estimate the 97.5 per cent VaR using two standard deviations of the sample return distribution. For long positions, Cr. buVaR is just this VaR multiplied by the inflator. Note that for short credit positions, the two risk measures are identical.

The results are shown in Figures 7–11. Generally, the ‘distress’ signal for Cr. buVaR, as represented by a sharp divergence between the two models, leads default events/credit stress by months. In effect, Cr. buVaR rapidly becomes larger than spread VaR whenever there is a RSW, but its increment is controlled so that the position can never lose more than its principal (less recovery value). In contrast, spread VaR is an inferior metric because it reflects only volatility; it is not impossible for spreads to trade at high
levels (reflecting high credit risk) but with low volatility if the market is illiquid and quotes change infrequently. In this case, credit risk will be understated.

**INTERPRETATION OF DEFAULT RISK**

What does the difference between Cr. buVaR and spread VaR represent? Consider the dashed curve in Figure 3. When the spread is at 20bp (AAA-rated range) the inflator is at 1.5, it implies that for the best issuers, a long credit position is 50 per cent riskier than a short credit position. When the spread is at 350bp (the average five-year CDS spread of Philippines in our data set), the inflator is at 3. For such ‘high yield’ issuers, a long credit position is three times riskier than a short credit position. For $\alpha_2 = 0.5$, the inflator is capped at 4.4 times. Is it reasonable to inflate the long side’s VaR multiple times more than the short side?
because of default risk? The author offers a heuristic argument.

The ‘credit spread puzzle’ is well researched; see Amato and Remolona. It refers to the fact that corporate credit spreads are much wider than what is implied by expected default rates calculated from historical default statistics of rating agencies. Studies by Elton et al. found that expected default rate accounted for no more than 25 per cent of observed spreads. Dionne et al. showed that results can differ widely depending on the methodology, default cycle and recovery rate assumption. In particular, for the period of high-default cycle, the proportion for Baa 10-year bonds can go as high as 71 per cent of estimated spread assuming a recovery rate of 49 per cent. For this case, when the recovery assumption is reduced to 40 per cent (a more conservative assumption), the proportion goes up to 83 per cent.

Hence, depending on the severity of the default cycle, the portion of default risk in credit spreads can go up to 83 per cent. The other 17 per cent is explained by volatility risk premium, liquidity premium and difficulty in diversification of tail risks. The author argues that a short credit position is free from default risk and, hence, should have only a non-default risk portion that can be as low as 17 per cent. A long credit position has both default and non-default risks. Heuristically, the ratio of risks between long and short positions can be inferred.

As a rough approximation, if up to 83 per cent of the spread is attributable to default risks (ie at least 17 per cent to non-default risks), then a long credit position can be up to 100/17 = 5.9 times riskier than a short position. It follows that the inflator should range from 1 to 5.9 depending on the extent of credit deterioration, in rough agreement with the cap of 4.4 times. In other words, the incremental loss of Cr. buVaR over spread VaR can be thought of as coming from default risk.

If this interpretation is accepted, then buVaR can be used to measure the risks of a credit trading portfolio comprehensively. Consider a portfolio consisting of a bond of issuer A and a CDS of issuer B. If the inflated scenarios of A spread is used to value the bond, and the inflated scenarios of B spread is used to value the CDS — and their P&L vectors are combined — then the portfolio buVaR will reflect the diversified risks of spreads, asymmetric credit default, basis between A and B, and credit correlation.

**IMPLICATIONS FOR SUPERVISION**

A capital regime based on Cr. buVaR will be more penal for long credit positions than for short credit positions. This is prudential because financial institutions will be systematically encouraged by punitive capital to dispose of bond holdings or buy CDS protection for issuers which are showing early signs of credit distress. Secondly, the Cr. buVaR typically rises months ahead of a default. This gives participants adequate time to trade out of positions and for risks to redistribute in the financial system. In contrast, spread VaR is always late and surprised by credit events. Thirdly, non-banks that are unaffected by capital rules can quote more competitively for holding long credit positions. This encourages a systemic transfer of credit risk away from the banking system to global investors and is macro-prudential. From the banks’ perspective, there is insignificant
counterparty risk exposure to clients since banks typically require clients to fully fund or collateralise their positions.

SUMMARY
The author introduced a new credit risk metric called Cr. buVaR which can be useful for the calculation of capital for the trading book. The method has distinct advantages:

1. Cr. buVaR captures the observed phenomena of credit risk asymmetry and pre-default RSW. By inflating the distribution on the positive side whenever credit distress is encountered, buVaR accounts for the skew/fat-tail risks understated by spread VaR.
2. Cr. buVaR is a forward-looking measure as it is conditional on spread level which reflects expectation of credit worthiness. Evidence from the 2008 credit crisis shows that RSW, and hence Cr. buVaR, often rises months ahead of actual/near defaults.
3. Unlike credit models, Cr. buVaR is not affected by procyclicality as it does not use credit ratings/transition matrices.
4. Cr. buVaR is able to integrate spread and default risks into a single metric, thus bypassing the problems of aggregation. It is straightforward to aggregate Cr. buVaR and general market VaR in a fully diversified way using an historical simulation approach.
5. An industry-wide application of Cr. buVaR for risk capital will likely be prudent for individual banks and the financial system.

The author emphasises that Cr. buVaR does not give a statistically precise solution — it is affected by the subjective choice of parameters (as are all VaR approaches) and the response function. Instead, it aims to provide a heuristic method that is more prudent for the purpose of regulatory risk capital.

DISCLAIMER
The views expressed in this paper are those of the author and do not represent the views of any organisations.

References