Market BuVaR: A countercyclical risk metric

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Abstract

The malfunction of the Value-at-risk (VaR) model is a risk management failure during the 2008 credit crisis. This metric is now criticized for being too little, too late. We propose an improvement—making VaR countercyclical and more robust to fat-tails. The new metric is called, bubble-VaR (BuVaR), the expected shortfall of a trading book portfolio removed of the effects of procyclicality. It involves inflating one side of the return distribution of an asset by a scaling factor called bubble that depends on the location of the present state in the boom-bust cycle. In a boom cycle, the negative side of the distribution is inflated, in a bust cycle, the positive side is inflated. Compared to VaR, BuVaR is countercyclical (it leads crashes), distinguishes between long and short positions (is asymmetrical) and provides an additional buffer for fat-tails by recognizing that crashes can happen only in the counter-trend direction. Thus, this method is useful for the purpose of a countercyclical capital buffer for market risk.

The approach relaxes the VaR assumptions of i.i.d. and stationarity of variables. It postulates that empirical phenomena of fat-tails, skewness, volatility clustering and the leverage effect, can be better understood by modeling the noise and cycle components together, instead of just the noise of the time series as modeled in VaR.

Keywords: Value at risk, procyclicality, extreme events, countercyclical capital buffer, market cycles, time series analysis

A longer explanation of some of the ideas of this paper and empirical results will be published in the author’s coming book “Bubble value at risk: extremistan and procyclicality”. The views expressed in this article are his own and do not represent the views of any organizations that he is affiliated to.
INTRODUCTION

The 2008 credit crisis was a costly stress test for the global banking system’s risk management preparedness. Critically, the popular value-at-risk (VaR) model used for the measurement of market risks was discredited for its gross underestimation of fat-tail risks during that stressful period. VaR is the de facto risk metric approved by the Basel for the calculation of regulatory capital under the internal model approach; its failure left regulators at loss for a workable alternative.

As discussed in Jorion \(^1\), the VaR of confidence level \(q\%\) is the \((1-q)\) quantile of the loss distribution estimated usually over a 1-day horizon. The weaknesses of VaR have long been forewarned; for example, see Danielsson et al. \(^2\). One flaw is its procyclical nature highlighted by the FSA’s Turner Review \(^3\), a response paper to the crisis. Procyclicality refers to the property of amplifying the cyclical tendencies of the market behavior. This is, firstly, an artifact of using a rolling window for VaR computation, which makes VaR always lag sharp market movements such as the sell-off at the onset of a crisis. Hence, VaR is perpetually late in crisis detection. Secondly, VaR also reflects the leverage effect, whereby price volatility is low during market euphoria and high during crises—this makes the capital regime business-friendly during a boom and punitive during recession. Thirdly, mark-to-market accounting practices also contribute to procyclicality by requiring banks to realize gains and losses immediately as they occur during a typical business cycle. Together, these three factors systemically encourage banks to invest/lend during an upturn, and cut-losses/call back loans during a downturn, thus amplifying the market cycle.

Another flaw of VaR is its inability to accurately estimate the extreme losses of fat-tailed distributions. Taleb \(^4\) argues that this is a result of the extremistan nature of financial markets. Extremistan refers to the a-typicality (or non-reproducibility) of certain phenomena which renders statistical estimation ineffective. Taleb \(^5\) found that most economic variables are patently fat-tailed with no known exceptions and are likely extremistan in nature. Without the element of reproducibility, statistical modeling of extreme tail losses is a futile endeavor and its results possibly misleading. As a practical solution, Taleb \(^5\) proposed a paradigm of focusing on “escaping” the tail risks (say by hedging) rather than adamantly measuring it. It uses a classification scheme called the fourth quadrant to identify situations where extremistan is dominant. The lack of reproducibility also brought into question the appropriateness of the use of frequentist statistics in the measurement of tail risk. Rebonato \(^6\) discusses this problem and advocates a Bayesian approach.

The Basel III framework just released in Dec 2010, included a countercyclical capital buffer for the banking system—requiring more capital during the boom phase which can be released and used as a loss buffer during a downturn, see BIS working paper \(^7\). However, the proposal is mainly for loans in the banking book. The new rules for market risk (see BIS revision \(^8\)) should generally produce a buffer a few times the current VaR-based capital in order to protect against the risk of fat-tail extreme events. The objective of this paper is to propose a countercyclical metric for the trading book capital regime.
We posit that the two mentioned failings of VaR originate from the assumptions underlying the construction of the VaR model. The need to infer the quantile estimate over the next time step (or horizon) necessitates the assumptions of i.i.d. and stationarity. These have the added advantage of making the model tractable. For example, in time series models, consistent and unbiased estimation of parameters can be done using known statistical methods such as regressions. Also, quantile measures will have correct probability interpretation i.e. a quantile number observed on a frequency distribution can be translated into a probability distribution. For example, the tenth largest loss in a distribution of 100 past observations can really be interpreted probabilistically as a level which 10% of future loss observations are expected to exceed. These assumptions are just a sleight of hand of modelers—empirical evidence has shown that such assumptions are routinely violated when a market is under stress.

Furthermore, if extremistan is present, such contrived abstractions (i.i.d. and stationarity) give rise to an illusion of precision in the measurement of extreme events which are by nature unpredictable (but largely avoidable in finance by hedging). Taleb argued this illusion of safety may undermine risk management preparedness.

The assumption of i.i.d. and stationarity requires that risks be modeled based on returns (as opposed to prices), moreover empirical research provides the comfort that returns exhibit these ideal characteristics during regular periods (even though VaR is more concerned with rare extreme losses where such assumptions do not hold). The process of differencing to arrive at returns means that useful information on price levels—such as the knowledge of the cyclical phase of the market, and microstructure—is lost from the VaR input.

This paper proposes the relaxation of these two assumptions and for the incorporation of the cyclical component of prices into the VaR model. We call this bubble VaR or “buVaR”. The insight behind this idea is the observation of asymmetry of crashes—when a crash occurs, it always crashes downward (never up). More generally, the sharp retracement we call a “crash” happens only in the countertrend direction. It follows that at the peak of a bubble, long positions are more risky (to a crash) than short ones, and at the trough of a downturn, short positions are more risky (to a bounce) than long ones. VaR is unable to reflect this seemingly intuitive risk asymmetry because there is no cyclical information in its input. Without this directional risk, VaR necessarily underestimates fat-tail losses.

CLASSICAL DECOMPOSITION, NEW INTERPRETATION

In time series analysis, the classical decomposition breaks the prices into three

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1 Regression models using prices will lead to serial correlation in residuals which renders estimation biased. Financial time series prices are mostly integrated to order I(1), meaning differencing once will lead to a stationary time series.

2 This is supported by behavioral evidence and studies by Bates and subsequent researchers that the put-call ratio, a known indicator of “fears of crash”, tends to rise near the market peak prior to major crashes.
components:

\[ X_t = L_t + S_t + Z_t(\varepsilon_t) \]  

where \( X_t \) is the original price series, \( L_t \) the long-term trend, \( S_t \) the cycle and \( Z_t \) the noise component. Of the three components, only \( Z_t \) is stationary and is assumed driven by an i.i.d. process \( \varepsilon_t \). \( L_t \) and \( S_t \) can be modeled using function-fitting and Fourier series, while \( Z_t \) is normally derived by taking first differences (or “detrending”). By working with returns, VaR deals exclusively with the distribution of \( Z_t \) and its quantile. So far VaR modeling has had limited success in capturing some of the well-observed market phenomena such as fat-tailness, skewness, volatility clustering and the leverage effect. We posit that this is because the distributional properties of \( Z_t \) such as moments and quantiles are missing essential information contained in the cycle \( S_t \).

We propose a new interpretation where the long-term trend is driven by real economic growth, cycles are caused by speculative excess (“bubbles”) and the noise component is the realization of trading under normal efficient market condition. The last can be modeled adequately by VaR. First, we posit that the fat-tail phenomenon is caused by a break in the cycle \( S_t \) which we identify as a market crash (or bubble burst). Crashes are just corrections in the cycle—asymmetric and often sharp. The leverage effect is explained by observing that a downward cycle break is more common than an upward cycle break. Secondly, we posit that a cycle compression increases serial correlation in the return series which in turn gives rise to volatility clustering. The notion is that cycles do not maintain a constant shape; in times of market stress the periodicity may shorten and the amplitude may increase. Figure 1 illustrates the new interpretation—a price series is synthetically constructed using its three components, cycle breaks are introduced as vertical falls in the cycle and cycle compression is introduced in the shaded zone. The three components are then recombinced and returns taken. The return series in the lower panel shows the manifestation of fat-tailness, skew and clustering effects—similar to what is observed in the marketplace.

The practical benefit of the new interpretation is that cycle formation and distortions of \( S_t \) can then be used to improve the VaR distributional forecast.

**FRAMEWORK FOR BUVAR APPROACH**

BuVaR starts with the premise that the tails of financial variables are extremistan. Hence, an exact measure of tail risks is not feasible—we strive only to be more accurate than VaR. The end goal is to obtain a practical model for capital charge that is countercyclical and buffers against fat-tail losses i.e. to robustify VaR.

The idea is to penalize (in terms of capital charged) any asset bubbles detected in the cycle \( S_t \) so as to “prevent” a crash. To do this we take the return distribution of \( X_t \) and inflate it in a direction that discourages bubble-chasing. The inflation is achieved by a multiplication factor called an inflator \( (\Delta_t) \). We also construct a market bubble indicator \( (B_t) \). So if the bubble forms in an uptrend, the negative side of the distribution is inflated (which would hurt the longs), if the bubble forms in a downtrend, the positive side of the
distribution is inflated (which would hurt the shorts). The long-term trend \( L_t \) is not penalized because regulators should not discourage participation in real economic growth.

Mathematically, on day \( t \), the return variable \( R_n = \ln(X_n/X_{n-1}) \) undergoes a transformation:

\[
R_n \rightarrow \begin{cases} 
\Delta_t R_n & \text{if } \text{sign}(R_n) \neq \text{sign}(B_t) \\
R_n & \text{if } \text{sign}(R_n) = \text{sign}(B_t) 
\end{cases}
\]  

(2)

where \( \Delta_t \geq 1 \) is the inflator, \( B_t \) is the bubble indicator, \( \Delta_t \) is a function of \( B_t \) and \( n \) is the scenario number in the historical simulation VaR approach.

BuVaR is not a probability weighting scheme—the inflator is not a probability weight. For example, an inflator of three applied on the left side of the distribution does not mean that losses are three times more likely than gains, or that we are three times more confident of measuring negative returns than positive ones. (Rebonato \(^{10}\) discusses the nuances of historical simulation VaR with non-constant weights.) Economically, the transformation compensates for asymmetric “crash risk” not well captured in the distribution of \( R_n \) and traditional VaR.

BuVaR is a framework to construct a countercyclical VaR for the purpose of capital buffer. It consists of three steps:

1. An indicator \( (B_t) \) to measure the formation of bubble, which we take as the degree of price deviation from an equilibrium level. The criteria for a good indicator are:
   i. The indicator must synch with (or ideally lead) the market cycle. This will provide the countercyclical property.
   ii. The indicator must not penalize investments during long periods of sustainable growth i.e. must differentiate between an unsustainable bubble \( (S_t) \) and a sustainable long term trend \( (L_t) \).
   iii. The indicator must persistently penalize positions which are against a market crash throughout a crash. Otherwise, it will encourage banks to “average down” or load-up positions in a falling market, an imprudent design.
   iv. The measure must be relatively stable for minimum capital use.

It is instructive to note that using price deviation from a simple moving-average (MA) will not work because it does not fulfill the criteria ii, iii and iv. See Figure 2.

2. A suitable inflator \( (\Delta_t) \) determined using a boundary argument—the loss is expected to be above VaR (a known underestimate) but below some structural upper limit, and logically increases with bubble formation. The criteria for a good inflator are:
   i. Monotonically increasing with \( B_t \)
   ii. Has a reasonable (or logical) upper limit. Structural upper limit exists because of imposition of circuit-breakers by regulators.
   iii. Can be calibrated to the asset’s own episodes of crashes and manias. It should not be a “one size fit all” multiplier for capital.
3. Computing a suitable tail risk measure after the transformation (2). We have chosen to use *expected shortfall* (see equation (6)) because it is:
   i. Coherent, in particular subadditive
   ii. Relatively stable for the purpose of minimum capital
   iii. Adequately responsive to new market prices and regime switches

**MODEL IMPLEMENTATION**

We derive an inflator using a procedure called *rank filtering* (commonly applied in digital signal processing) which can be summarized by the following steps:

1. Derive a vector of daily returns $R_n = \ln(X_n / X_{n-1})$ for all previous $n$ days.

2. Apply an 8% rank filter to $R_n$ of the past 1000-day rolling window\(^{iii}\). This means that we make $R_n=0$ whenever it is below 8% quantile\(^{iv}\) or above 92% quantile of a rolling distribution $\{R_n,..,R_{n-1000}\}$. In effect, the exceedences are removed.

3. Create a vector of growth factors given by: $D_n=\exp(R_n)$ for each $n$.

4. At each day-$n$, reconstruct a 1000-day new price vector $\{P_n,..,P_{n-1000}\}$ “backward” iteratively using the “starting” price $X_n$. In other words, $P_n = X_n$ (the starting price) then calculate $P_{n-1} = P_n / D_n$ iteratively until $P_{n-1000}$. This vector is called the *alternate history* where market bubbles and manias did not exist hypothetically and growth was gradual and sustainable.

5. We define the *equilibrium* $\mu_n$ as the $m$-day moving average of the alternate history, where $m$ is non-constant and is given by:

\[
m = \text{Int}[\text{Min}\left\{ \frac{\text{Stdev}(X_n, X_{n-1},..,X_{n-500})}{\text{Stdev}(X_n, X_{n-1},..,X_{n-1000})} \times 1000,1000\right\}] \tag{3}
\]

This *adaptive* moving average has the effect of reducing the *bubble* (by shortening the window length $m$) once a strong rally/break is found to be sustainable and is deemed due more to $L_t$ than to bubble formation, consistent with not penalizing long-term growth. Figure 3 compares the *bubble* indicator derived by averaging with equal weights (simple MA) and adaptive weights.

\(^{iii}\) The last three major crises (2007, 1997, 1987) occurred 10 years apart, so the window length must be shorter than 10 years in order to exclude distortion from previous crises. Yet, it must be long enough to contain the most recent cycle. A 1000-day window length equivalent to four trading years is found to be workable by the author.

\(^{iv}\) Empirical study by the author shows that a threshold around 8% is workable. At higher quantiles, too much information is filtered from the data that the resulting *bubble* flattens and loses its cyclicality. But if the threshold approaches zero, our method reduces to a deviation from a simple moving-average (which as we mentioned earlier will not work).
6. We define the bubble indicator as the price deviation from the equilibrium:

\[ B_n = X_n / \mu_n - 1 \]  

(4)

It measures the degree of cyclical bubble formation, and unlike simple MA deviation, meets the criteria set out in the previous section. Figure 2 compares the two results.

7. A choice of inflator which satisfies the criteria outlined in the previous section is:

\[ \Delta_t = \text{Min} \left( \frac{\Psi}{2\sigma_i}, \exp \left( \left( \frac{\text{Abs}(B_n)}{B_{\text{max}}} \right)^{\omega_2} \ln \left( \frac{\Psi}{2\sigma_i} \right) \right) \right) \]  

(5)

where:

- \( \Psi \): average of 5 largest losses and gains in all history of that asset (expressed in absolute terms), capped by a circuit-breaker if applicable.
- \( B_{\text{max}} \): largest absolute \( B_n \) observed in all history of that asset
- \( \sigma_i \): standard deviation of returns of the last 250 days
- \( \omega_2 = 0.5 \)

The derivation of (5) and choice of \( \omega_2 \) are explained in Appendix A.

The inflator \( \Delta_t \) (≥1) is a multiplicative adjustment to every scenario on one side of the return distribution of a 250-business-day observation period (Basel stipulates the use of a minimum 12-month observation period for VaR). As per equation (2), on day \( t \) if \( B_t > 0 \), multiply every scenario of the negative side to penalize long positions (but set \( \Delta_t = 1 \) for all positive returns). If \( B_t < 0 \), multiply the scenarios on the positive side to penalize short positions (but set \( \Delta_t = 1 \) for all negative returns).

BuVaR is based on a historical simulation approach, where a portfolio is evaluated at “shifted” levels \( (X'_i) \) based on a set of scenarios. To ensure the shifted levels for the asset (or risk factor) do not become negative (so that \( X'_i \geq 0 \)), they should be calculated using log-returns:

\[ X'_i = X_i \exp(R_n \Delta_t) \]  

(5)

where scenario \( i=1, 2, \ldots, 250 \) is the number of days to the past from day-\( t \), \( R_n \) is the original return series (prior to rank filtering), and \( n = t-i+1 \). For this uni-variate case, a bank’s portfolio is first evaluated using a product pricing function \( g(.) \) at the current state \( X_t \) giving value \( g(X_t) \). Then, for each scenario-\( i \) the portfolio is evaluated at state \( X'_i \) to give a value \( g(X'_i) \). The P&L vector at day-\( t \) is just the distribution of values \( \{g(X'_i) - g(X_t)\} \), where \( g(X'_i) \) is a 250-vector and \( g(X_t) \) a scalar. Let the sample distribution of this P&L be \( y \).
The buVaR at confidence level $q\%$ is the expected shortfall of the P&L distribution $y$ estimated over a 1-day horizon at $(1-q)$ coverage:

$$BuVaR_q = E(y \mid y < \mu) \text{ where } \Pr(y < \mu) = 1 - q$$

(6)

By using expected shortfall, we ensure that buVaR is coherent (in particular, sub-additive) even though the inflated joint distribution is certainly non-elliptical.

**VISUAL TESTING**

The buVaR method is non-stationary since the day-to-day distribution is multiplied by an inflator that changes daily. Also, buVaR generally leads market crashes; that means statistical back-testing (which requires the P&L and risk measure to be contemporaneous) is not appropriate—we will need to rely on visual testing of the results.

We compared buVaR against conventional expected shortfall using daily data between Jan 1990 and Jun 2010 for the S&P 500 index (source: Bloomberg Finance L.P.). The expected shortfall is taken at 97.5% confidence level for a rolling 250-day observation period. As per Figure 5, BuVaR leads expected shortfall (or any convention VaR measure) significantly and peaks almost one year ahead of the 2008 crash.

Figure 2 shows the bubble measure for the S&P 500 Index. The bubble peaked at the end of 1995 but the rally was sustainable and developed into a long-term uptrend that lasted until mid-2000. Due to the use of adaptive weights, the bubble reduced but maintained a moderately positive level up to 1999. Generally, the bubble will peak and trough in tandem with market prices. Note that the bubble did not peak in 2000 for the S&P 500. This is because the dot com crash happened to the Nasdaq index and not to the S&P 500, which entered a bear market in a more gradual way. Further results (Figure 6) show that the internet bubble burst in 2000 coincided with the peak in buVaR for the Nasdaq index.

Comprehensive tests for buVaR are done in Wong 11, covering a range of assets including: S&P 500 index, Nasdaq 100 index, Nikkei 225 index, Hang Seng index, USD/JPY rate, AUD/SGD rate, USD 10-year swap, USD 5-year swap, USD 3-month Libor, gold spot and crude oil futures. The results are characteristically consistent with the conclusions of this paper.

Analysis indicates that buVaR is reliable for risk factors that are based (quoted) on price and rates. However, it is unsuitable for (option) volatility-based risk factors because they tend to be more stationary; without trends nor cycles, there is no need for buVaR—conventional VaR (or setting inflator to 1) suffices. BuVaR is also unsuitable for credit spread risk factors because spreads exhibit mean-fleeing behavior when there is credit deterioration, an idiosyncratic risk not related to market cycles. Credit spread risk is modeled in a separate paper by Wong 12 using an extension of the buVaR approach.
SUMMARY

This paper introduces a method to robustify VaR for the purpose of market risk capital calculation for banks. The buVaR method has distinct advantages:

1. It is overtly countercyclical: unlike VaR, it is one step ahead of the crisis (or crash), often buVaR peaks months in advance.
2. It recognizes risk asymmetry, that the risks of long and short positions are unequal. It penalizes (discourages) positions that are “chasing the bubble”.
3. It provides capital buffers for fat-tail events: when an asset bubble is forming, buVaR becomes a multiple of the VaR and forms a buffer against the risk of a crash in the corrective direction.

However, the relaxation of the i.i.d. and stationary assumptions meant the loss of tractability, estimation consistency (in a statistical sense) and precision (reproducibility). We argue that such assumptions are violated anyway under stressful market conditions. BuVaR is a more accurate risk metric than VaR, in that it gives a “best guess” of the estimated loss between VaR (which is known to understate risks) and a reasonably estimated upper bound. We stressed that buVaR does not give a single mathematically precise solution, rather, it provides a workable number that is useful for the purpose of risk capital. The higher capital buffer compensates for our lack of knowledge of fat-tail phenomena.

Since buVaR requires the use of daily price data, its application is limited to the trading book capital, or for market risk. It is conceivable that systemic application may have the effect of dampening the market cycle because positions that are “chasing the bubble” become increasingly costly (in terms of regulatory capital) compared to contrarian positions.

BuVaR is also a departure from conventional frequentist thinking. It adopts a perspective that extreme tails are inherently immeasurable. Questions of “true risk” are empty, and a risk metric’s value lies in its usefulness. The crucial question to ask is: used in a given application, will buVaR promote prudence and desirable behavior at financial institutions?

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v We emphasize the distinction between accuracy, a degree of authenticity, and precision, a degree of reproducibility.
References


**Figure 1:** Illustration of the *new interpretation*
Breaks are introduced into the cycle component (vertical falls), cycle compression is also introduced in the shaded zone. The lower panel shows the resulting return series is manifestedly fat-tailed, skewed and serially correlated (which leads to volatility clustering).

**Figure 2:** The equilibrium and *bubble* measure for S&P 500 index
The *bubble* is a superior to using simple 1000-day moving average (MA). It penalizes longs *throughout* the 2008 crisis and does not change direction half-way into the crash like the MA deviation. The *bubble* declined from 1995-1999 when the rally was sustained, while the MA deviation continues to rise. The *bubble* is always smoother than the MA deviation.
Figure 3:  *Bubble* indicator derived using simple MA and adaptive MA

![Figure 3: Bubble indicator derived using simple MA and adaptive MA](image)

Figure 4:  Inflator response function of different $\omega_2$ (where $\sigma=2.5\%, B_{max}=6, \Psi=10\%$)

![Figure 4: Inflator response function of different $\omega_2$](image)
Figure 5: BuVaR and estimated short fall for S&P 500 index
(with $\omega_2=0.5$, $B_{max}=0.34$, $\Psi=10\%$)

Figure 6: BuVaR and estimated short fall for Nasdaq index
(with $\omega_2=0.5$, $B_{max}=0.34$, $\Psi=10\%$)
Appendix A  Derivation of the Inflator Response Function

The purpose of the inflator is to increase the risk measure in proportion to the degree of bubble formation $B_t$. We propose the following response function:

$$\Delta_t = \exp[\omega_1 B_t^{\omega_2}]$$  \hfill (A1)

where $\omega_1$, $\omega_2$ are positive parameters. We need to cap $\Delta_t$ so that buVaR does not grow without bound. This cap is defined as the inflator that will inflate the VaR (which we simply take as $2\sigma_t$) to $\Psi$, an upper bound which can be set by the regulator. BuVaR will be more conservative (or penal) with a higher $\Psi$ setting.

$$\Delta_{Max}(2\sigma_t) = \Psi$$  \hfill (A2)

The idea of placing an upper bound $\Psi$ on VaR is not unreasonable. Given the circuit-breakers (typically set at 10%) put in place by exchanges nowadays to contain uncontrolled single day losses, we should not expect to see extreme events beyond such a magnitude (unless the shock is of a non-market nature, in which case is outside the scope of market VaR). By arbitrage argument, OTC products that have hedgeable equivalents on exchanges will also be restrained by such circuit breakers.

The upper bound has to give some head-room for the occurrence of fat-tail events but setting $\Psi$ too high (say 25% to cover a 25% daily loss as occurred in the 1987 crash) will not work, as it will be too uneconomical for banks to be perpetually braced for disaster during normal times. A compromise is to set $\Psi$ such that the resulting buVaR is slightly higher than the expected shortfall witnessed during the 2008 crisis, but where this buVaR will be delivered (or charged) ahead of crises during the euphoria (run-up) phase. A workable setting is to define $\Psi$ as the average of five largest losses and five largest gains in all history of the asset (in absolute daily returns), capped by a circuit-breaker if applicable.

We then relate this upper bound to the largest observed bubble $B_{max}$ in all history. In other words:

$$\Delta_{Max} = \exp[\omega_1 B_{max}^{\omega_2}]$$  \hfill (A3)

By eliminating $\omega_1$ the capped $\Delta_t$ then becomes:

$$\Delta_t = \min \left\{ \frac{\Psi}{2\sigma_t}, \exp \left( \frac{Abs(B_t)}{B_{max}}^{\omega_2} \ln \left( \frac{\Psi}{2\sigma_t} \right) \right) \right\}$$  \hfill (A4)

The parameter $\omega_2$ tunes the curvature of the response function in Figure 4. The author’s study shows that $\omega_2=0.5$ provides the smoothest day-to-day variation in buVaR and is relatively stable for the purpose of risk capital.